TOPOLOGICAL SIMPLIFICATIONS FOR MECHANICAL VIBRATION ANALYSIS

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by

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ABSTRACT

A theoretical method for representing the physical characteristics of a portion of a mechanical piping system in terms of a stiffness matrix involving only those joints at which the sub-system is connected into the overall system is presented herein. The use of this method in reducing the size of the matrices that are needed for finding the normal vibration frequencies of a piping system is discussed, and an outline for a digital computer program to use this method is presented.

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CHAPTER I

INTRODUCTION

1.1 General Remarks

There are many aspects to be considered in the design of any piping system; stresses, strains, pressure drops, and corrosion, to name a few. This thesis deals with only one aspect of the overall problem, that of the dynamic behaviour of the system as represented by its natural frequencies of vibration, and presents a method whereby a large system may be systematically attacked by considering small sub-systems in order to improve computer utilization.

Except for very simple or trivial problems the number of calculations involved and the size of the matrices used make the use of a large digital computer mandatory. No digital computer program has been written, however, but an outlined plan of approach is included in this thesis.

1.2 Scope of Work Presented

Baird [1]* has presented a method for determining analytically the undamped natural frequencies and mode shapes of large three dimensional piping systems by use of stiffness matrices. He also has made use of topological matrices to express mathematically certain physical or boundary conditions. The most obvious inherent disadvantage of this method, however, is the large size of the matrices which may be required with the result that supplementary memory of some sort must be

*Numbers appearing in this manner refer to the bibliography, page 41.

used in the computer solutions, thus increasing the computation time considerably.

A technique is presented in this thesis whereby small portions of the system, called ganglia or sub-systems, can be analyzed and reduced to some sort of pseudo-elements having certain calculated characteristics with the result that only the reactions at the ends of these pseudo-elements are of interest, and all the internal characteristics are engulfed in the general stiffness matrix for each ganglion. These individual sub-systems can be used to build up an entire system whose vibrational characteristics can be represented far more compactly than can those of the unsimplified system.

An outline for a proposed computer program to utilize this method is included in Chapter IV.

1.3 Definitions

A number of definitions must be established and the reader is warned that they are somewhat at variance with those of Baird [1]. The nomenclature used (see section 1.4) will also be at variance with Baird's.

Element - A piece of the system having two, and only two, ends and no internal branches. Normally an element will be a simple length of pipe. A tee, though, which is a more complicated item, should be thought of as three elements joined at a common point. Other items of piping hardware may be analyzed similarly.

Universe - This term is used in the same sense in which it is used in statistics, namely the larger body of effects from which the system is taken. In this case the universe for a piping system consists of all the boundary forces and constraints, foundations,

etc. which affect the system. When a sub-system is isolated, its universe is the larger system itself.

Node - A terminal point of an element is called a node. It is noted that a node is a theoretical point and not a physical item. There must be at least one element end at each node, though there may be any number greater than one. Nodes may be further classified as:

1. Trivial nodes - Those nodes upon which only two element ends are incident.

2. Boundary Nodes - Those nodes at which there are forces and constraints applied to the system by the universe. For purposes of convenience when dealing with coordinate transformations it is required that only one element be incident upon a boundary node. In many cases this may require the introduction of an element connecting a primary node to the boundary node at which the system connects to the universe. These primary and boundary nodes may have the same geometrical location, though they are topologically distinct.

3. Primary Nodes - All nodes of the system that are not trivial or boundary nodes.

Path - A sequence of elements connecting two nodes such that there are only trivial nodes connecting these elements.

Primary Path - A path connecting two primary nodes or a primary node and a boundary node. A primary path may contain only trivial nodes on its interior.

Primary Element - An element made up of all the elements on a primary path.

State Vector - A column matrix of displacements and corresponding forces at any point. Since in general it requires three translations and three rotations to describe motion completely the state vector is a 12 x 1 matrix. For the purposes of this thesis it has been convenient to make the first six elements of the column the displacements in the six independent degrees of freedom, and the last six elements the corresponding forces.

Transfer Matrix - A square (12 x 12) matrix representing the physical characteristics of an element such that the state vector at one end will equal the product of the transfer matrix and the state vector at the other end, in that order.

Dynamic Transfer Matrix - A transfer matrix that takes into account the mass of the elements and the frequency of vibration in addition to the static properties (i.e., Young's modulus, Poisson's ratio, second moments of area, etc.).

Ganglion - A sub-system "removed" from the system at primary nodes. "Removed" is used to indicate that the ganglion is considered as a separate system whose boundary nodes are the nodes at which it was connected into the overall system.

1.4 Nomenclature

A capital letter will normally be used to represent a matrix and appropriate sub- and super-scripts may be used to further define it as follows:

$$\begin{matrix} a & & c \\ b & A & d \end{matrix}$$

a - number of rows

b - number of columns

c - may be 1) blank, 2) T indicating the transpose of the matrix, or 3) -1 indicating the inverse of the matrix

d - used for identification and described as required in context.

Brackets, [ ], may be used to delineate a matrix or the partition of a matrix if the meaning is thereby clarified. They will always be used when the elements of a matrix are exhibited *in extensio*. Braces, $\{\ \}$, will be used to permit the writing of the terms of a column vector as a row vector to save space. The term Diag$\{\ \}$ will mean that the n terms inside the braces are the n diagonal terms of an otherwise null matrix, though the terms themselves may represent square sub-matrices.

In no case will more than one capital letter be used for a matrix; thus the expression AB will mean the product of matrices A and B in that order and will imply that A is conformable to B for multiplication. One should recall that matrix multiplication is distributive and associative though not necessarily commutative.

All nodes will be numbered, and for convenience the boundary nodes will be given the lowest numbers. The ends of the elements will be identified by lower case English letters, two per element.

Notation generally follows that of Pestel and Leckie [2]. The following are the matrix symbols used.

A Node incidence matrix

D, F Displacement and force vectors (or partitions) respectively

G Coordinate rotation matrix

K Dynamic stiffness matrix

K_N Node stiffness Matrix

$|\bar{K}_N|$ Frequency determinant

K_p Primitive stiffness matrix

T Transformation matrix

U Transfer matrix

Z State vector

By the use of subscripts any of the above symbols may be used to represent an element or partition of the full matrix, thus $Z_{1,2}$ would be an element or partition (depending upon the context) of matrix Z, namely the one appearing in row 1, column 2.

The following scalar quantities are used:

B The number of boundary nodes of a system

M/2 The number of primary paths in a system

(Therefore M is the number of primary element ends in the system.)

N The number of nodes (boundary and primary) in a system

The use of a prime, ('), will generally indicate that the quantity designated refers to a sub-system.

CHAPTER II

THE TRANSFER AND STIFFNESS MATRIX SOLUTION

A brief summary of the solution by use of stiffness matrices for the characteristic (normal) frequencies of a piping system with general topology is given in this chapter in order to refresh the reader's memory. For further details of the theory see Baird [1], but again, be aware of notational differences.

Before we discuss the dynamic stiffness matrix approach as developed in [1] and [2] it is necessary to remind the reader that this direct mathematical approach deviates somewhat from the approach to planning a digital computer program because, although on paper we can manipulate symbols, in the computer we must deal with numbers. It is assumed that the user of this method has available the necessary dynamic transfer matrices for whatever fundamental elements are involved and can compute (in one way or another) the geometrical matrices necessary to relate all quantities to some single coordinate system.

a. Calculate a transfer matrix for each primary element, eliminating the trivial nodes along the path. Thus:

$$U_{k,m} = U_{n,m}G_nU_{n-1,n}G_{n-1} \cdots U_{1,2}G_1U_{k,1}$$

where k and m are element ends at primary and/or boundary nodes, and n, n-1, ...1 are the trivial nodes along the path. G_n, G_{n-1},...G_1 are the coordinate rotation matrices providing the necessary coordinate alignment at each node. The resultant transfer matrix, $U_{k,m}$ is in the coordinate system of the element end at the node at which the path starts,

end k in this case. The relationship between k and m is now:

$$Z_m = U_{k,m} Z_k \qquad 2.1$$

b. Transform the transfer matrix of each primary element into a dynamic stiffness matrix as indicated in what follows. The user is cautioned to establish and adhere to an order within his state vector. The order used by Baird [1] is recommended. Referenced to his local coordinate system it is:

$$Z = \begin{bmatrix} \text{Displacement in an x direction} \\ \text{Displacement in a y direction} \\ \text{Displacement in a z direction} \\ \text{Rotation about an x axis} \\ \text{Rotation about a y axis} \\ \text{Rotation about a z axis} \\ \text{Force in an x direction} \\ \text{Force in a y direction} \\ \text{Force in a z direction} \\ \text{Moment about an x axis} \\ \text{Moment about a y axis} \\ \text{Moment about a z axis} \end{bmatrix}$$

Partitioning the state vectors into force and displacement vectors, and the dynamic transfer matrix into four 6x6 matrices yields:

$$\left[\begin{array}{c} D_m \\ \hline F_m \end{array}\right] = \left[\begin{array}{c|c} U_{11} & U_{12} \\ \hline U_{21} & U_{22} \end{array}\right] \left[\begin{array}{c} D_k \\ \hline f_k \end{array}\right] \qquad 2.2$$

An explanation of the use of the lower case f will follow shortly. The above equations then can be written:

$$D_m = U_{11} D_k + U_{12} f_k$$

$$F_m = U_{21} D_k + U_{22} f_k$$

Solving the first of these for f_k gives

$$f_k = U_{12}^{-1} D_m - U_{12}^{-1} U_{11} D_k$$

and substituting this into the second gives

$$F_m = U_{21} D_k + U_{22}(U_{12}^{-1} D_m - U_{12}^{-1} U_{11} D_k)$$

Reassembling these last two equations in matrix form, we have

$$\begin{bmatrix} -f_k \\ F_m \end{bmatrix} = \begin{bmatrix} U_{12}^{-1} U_{11} & -U_{12}^{-1} \\ U_{21} - U_{22} U_{12}^{-1} U_{11} & U_{22} U_{12}^{-1} \end{bmatrix} \begin{bmatrix} D_k \\ D_m \end{bmatrix} \qquad 2.3$$

There is a sign convention that must be carefully delineated at this time. In equation 2.1 the force elements of the state vector, Z_k, (shown in 2.2 as the partitioned sub-matrix f_k) are the forces applied <u>to</u> element km by the node at k, whereas the force elements of the state vector Z_m are the forces applied <u>by</u> element km to the node at m. It is obvious then that the negatives of the elements of sub-matrix f_k represent the forces applied <u>by</u> element km to the node at k; and if we define a new vector $F_k = -f_k$, then both F_k and F_m represent forces applied by the element to the nodes. Then equation 2.3 can be rewritten:

$$\begin{bmatrix} F_k \\ F_m \end{bmatrix} = K_{k,m} \begin{bmatrix} D_k \\ D_m \end{bmatrix} \qquad 2.4$$

where $K_{k,m}$, the dynamic stiffness matrix for the element km, is the square matrix in 2.3.

c. The dynamic stiffness matrix of each primary element is now known in the coordinate system of the element end at which calculations started. In order to relate all these matrices to each other it is necessary that some global coordinate system and reference axes be chosen and that all the dynamic stiffness matrices be transformed into this system. Detailed discussion of the necessary geometric matrices and multiplications may be found in [1] and [3] and will not be repeated here.

d. The system now consists of:

M/2 primary nodes

B boundary nodes

N nodes (boundary and primary)

zero trivial nodes

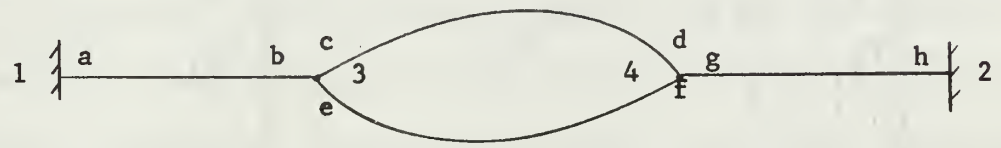
e. A topological matrix, A, relating the incidence of element ends on the nodes, is now generated as follows. Each of its elements is in actuality a 6 x 6 matrix, O or I, O representing a 6 x 6 null matrix and I representing a 6 x 6 identity matrix. The topological matrix will be generated in its transposed form, namely A^T, since both forms are needed in subsequent calculations. A^T will be 6N x 6M in size. The following rules obtain:

$A^T_{i,j}$ will be I if end j is incident on node i.

$A^T_{i,j}$ will be O if end j is not incident on node i.

I and O are 6 x 6 matrices as defined above. To permit doing this, a certain order must be established both for the nodes and the element ends, and this order will have to be adhered to for subsequent assemblages

and calculations. It will be helpful if the low numbered nodes (i.e., the upper rows of the matrix A^T) are the boundary nodes as this will minimize subsequent reordering. As an example, A^T is shown for the simple system given below.



Nodes are numbered, element ends are lettered. Nodes 1 and 2 are boundary nodes, nodes 3 and 4 are primary nodes.

$$A^T = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \left[\begin{array}{cccccccc} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \\ 0 & I & I & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & I & I & 0 \end{array}\right] \end{array}$$

Row of all element ends

Column of nodes, note the boundary nodes listed first

A^T is the quantity within the brackets. The row of element ends and the column of nodes are shown only to indicate how the matrix is generated.

As implied herein, A is $[A^T]^T$.

f. All the primary element dynamic stiffness matrices are now assembled into a large matrix called the primitive stiffness matrix, K_p. It is necessary that the order of assembly correspond to that of A^T.

$$\begin{bmatrix} F_a \\ F_b \\ \hdashline F_c \\ F_d \\ \hdashline \vdots \end{bmatrix} = \left[\begin{array}{c:c:c} K_{ab} & 0 & \cdots\cdots \\ \hdashline 0 & K_{cd} & \cdots\cdots \\ \hdashline & & \end{array}\right] \begin{bmatrix} D_a \\ D_b \\ \hdashline D_c \\ D_d \\ \hdashline \vdots \end{bmatrix} \qquad 2.5$$

Defining $\{F_a\ F_b\ \ldots\}$ as F_E (subscript E referring to ends of elements) and $\{D_a\ D_b\ \ldots\}$ as D_E then

$$F_E = K_p\ D_E \qquad 2.6$$

where K_p is the large square matrix in equation 2.5.

g. The sum of the forces exerted on any node by the element ends incident upon it must be zero (except for boundary nodes where the constraining forces are the "closing" forces) and the displacements of all the ends incident upon any given node must be identical. The topological matrices A and A^T permit these conditions to be expressed mathematically. Using the subscript N to refer to nodes, just as subscript E refers to ends of elements,

$$F_N = A^T F_E$$

and

$$D_E = A\ D_N.$$

F_N is a column vector of the forces exerted on the nodes and D_N is a column vector of the node displacements. Therefore:

$$\begin{aligned} F_N &= A^T F_E \\ &= A^T (K_p\ D_E) \\ &= A^T K_p\ A\ D_N. \end{aligned}$$

Defining $A^T K_p A$ as the node stiffness matrix, K_N, we then have the relationship between node forces and displacements:

$$F_N = K_N D_N \qquad 2.7$$

h. F_N is a column of N 6x1 matrices and all but B of these will have all the elements identically equal to zero. F_N, K_N and D_N are now reordered so that the B uppermost 6x1 matrices of F_N and D_N are the force and displacement vectors respectively of the B boundary nodes. This step is unnecessary if the proper order has been previously established as indicated in 2.e. Partitioning these matrices now yields:

$$\begin{bmatrix} {\scriptstyle 6B\times 1}\ F_{NB} \\ {\scriptstyle 6(N-B)\times 1}\ 0 \end{bmatrix} = \begin{bmatrix} K_{N11} & K_{N12} \\ K_{N21} & K_{N22} \end{bmatrix} \begin{bmatrix} D_{NB} \\ D_{N2} \end{bmatrix}$$

i. Since boundary constraints are known in the local coordinate systems of the various boundary nodes we now transform back to local coordinates. Up to this point all reordering has been done using entire 6x1 or 6x6 submatrices. This has been done in order to preserve the integrity of the partitions of the state vector so that we can make the transformation back to local coordinates by using the same transformation matrices generated earlier to transform the system to global coordinates. Now with F_{NB} and D_{NB} transformed to local coordinates we may find, upon examination of the boundary conditions, that some of the elements of F_{NB} are also equal to zero. Reordering the matrices again, this time element by element instead of by submatrices, so that the non-zero elements of F_{NB} and the corresponding zero elements of D_{NB} occupy the leading positions in their respective vectors, and then partitioning the

matrices we find:

$$\begin{bmatrix} {}^{*}F_{NB} \\ 0 \end{bmatrix} = \begin{bmatrix} {}^{*}K_{N11} & {}^{*}K_{N12} \\ {}^{*}K_{N21} & {}^{*}K_{N22} \end{bmatrix} \begin{bmatrix} 0 \\ {}^{*}D_{N2} \end{bmatrix} \qquad 2.8$$

${}^{*}F_{NB}$ is a Px1 vector where P is the number of non-zero boundary conditions and is less than or equal to 6B. The asterisk is used to indicate local coordinates and the reordering indicated in the text.

From equation 2.8 it can be seen that:

$$\begin{bmatrix} 0 \end{bmatrix} = \begin{bmatrix} {}^{*}K_{N21} \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} + \begin{bmatrix} {}^{*}K_{N22} \end{bmatrix} \begin{bmatrix} {}^{*}D_{N2} \end{bmatrix}$$

Since the elements of ${}^{*}D_{N2}$ are not zero, and are independent it appears that ${}^{*}K_{N22}$ must be singular, i.e.,

$$\left| {}^{*}K_{N22} \right| = 0 \qquad 2.9$$

Defining $\left| {}^{*}K_{N22} \right|$ as the frequency determinant, and calling it $\left| \bar{K}_N \right|$ for convenience, we can see that it will be of order (6N-P) x (6N-P). Since some of the elements of $\left| \bar{K}_N \right|$ are frequency dependent it can be seen that there will be some values of frequency for which the frequency determinant becomes zero. These frequencies will be the natural frequencies of the system. We are unable to solve for these frequencies directly since it would not be feasible to obtain a literal expression, ultimately expressible as a polynomial (or worse) in the unknown frequencies. Therefore we calculate the value of the frequency determinant for many different values of frequency (using a high speed digital

computer) and plot the value of the determinant versus the frequency. From this plot we should be able to pick, at least approximately, the natural frequencies. It is then possible to refine these answers by making additional calculations over much smaller intervals and plotting these results on an expanded scale. If a lumped mass model of the system has been used throughout, there will be a finite number of natural frequencies. If however, a distributed mass model has been used then there will be an infinite number of natural frequencies.

j. It can easily be seen that the sizes of the matrices involved in this method may become quite large. The largest single matrix will be K_p, which is 6M x 6M, and even for a system of only fifteen primary elements, this matrix would be 180 x 180. A matrix this size has 32,400 elements and if stored in the core memory of a 32K computer would leave only 368 cells for other variables, the monitor, the resident program, etc. This is far too few cells for these purposes. If double precision arithmetic were to be used then twice as many cells would be required and the matrix could not be stored in core memory.

CHAPTER III

REDUCTION OF SUB SYSTEMS

3.1 Purpose

It is the purpose of this thesis to develop and exhibit methods by which the mathematical manipulations may be simplified or partitioned with the particular purpose of keeping well within the core memory capacity of the usual large engineering computer. Matrices too large for the core can be handled with the use of auxiliary memories or intermediate tapes; but this slows computations by several orders of magnitude and is to be avoided if possible, particularly in a problem of this type where the solutions are gained by successive iterations.

3.2 Theory

The basic propositions which will be demonstrated are; (1) that any ganglion or sub-system can be represented by an equivalent stiffness matrix of size 6B' x 6B' where B' is the number of nodes at which the ganglion is connected to the overall system, and (2) the size of the overall system frequency determinant can be reduced so that it is, at most, 12B x 12B where B is the number of boundary nodes of the system.

Start by taking any ganglion of the system that does not include a primary element connected to a boundary node, and treat it as if it were a complete system having B' boundary nodes, N' nodes, and M'/2 primary elements. Going through the manipulations indicated in part II the equation representing this ganglion eventually may be reduced to the form:

$$F_{N'} = K_{N'} D_{N'} \qquad \text{(cf. equation 2.7)}$$

Then by ordering the matrices so that the B' boundary node vectors appear first in F_N and D_N and partitioning we obtain:

$$\begin{bmatrix} F_{NB'} \\ F_{N2} \end{bmatrix} = \begin{bmatrix} K_{N11} & K_{N12} \\ K_{N21} & K_{N22} \end{bmatrix} \begin{bmatrix} D_{NB'} \\ D_{N2} \end{bmatrix} \qquad 3.1.1$$

Now it must be understood that there is a difference between this case and the solution of the overall problem, the difference being that while the elements of F_{N2} are identically equal to zero, the elements of $D_{NB'}$ are not. In other words, for the overall system the existence of a boundary force indicates a rigid constraint component (i.e., no displacement) whereas for the sub-system, there is no such restriction. Proceeding,

$$\begin{aligned} F_{NB'} &= K_{N11}\ D_{NB'} + K_{N12}\ D_{N2} \\ 0 &= K_{N21}\ D_{NB'} + K_{N22}\ D_{N2}. \end{aligned} \qquad 3.1.1$$

Solving the second of these for D_{N2} we get

$$D_{N2} = -K_{N22}^{-1}\ K_{N21}\ D_{NB'}$$

Substituting it into the first equation yields

$$\begin{aligned} F_{NB'} &= \left[K_{N11} - K_{N12}\ K_{N22}^{-1}\ K_{N21}\right]\ D_{NB'} \\ &= K_{N'}\ D_{NB'} \end{aligned} \qquad 3.1.2$$

where $F_{NB'}$ and $D_{NB'}$ are 6B' x 1 and $K_{N'}$ is 6B' x 6B'. $K_{N'}$ is defined by this relation.

What has been generated might be thought of as a B'-ended pseudo-element to replace the ganglion and retain its identical characteristics. The pseudo-element is represented by a stiffness matrix, but note

that this is no longer the 12 x 12 stiffness matrix of a simple element but a 6B' x 6B' matrix.

This procedure may be repeated for other ganglia which are separate from those considered previously, or a pseudo-element and its associated stiffness matrix (or indeed several such elements) may be used as a building block in constructing a larger ganglion. Each pseudo-element calculation eliminates N'-B' nodes from the sub-system. This leads to the conclusion that the most "efficient" ganglion to choose is the one with the fewest boundary nodes for the most internal nodes.

With the various ganglia reduced to pseudo-elements, it is then possible to proceed as in Chapter II. A primitive stiffness matrix is assembled and pre-and post-multiplied by the topological matrices to yield a node stiffness matrix. This is reordered, transformed, and reordered again, the frequency determinant is isolated, and the natural frequencies are found by an iterative technique as described in section 2.i. What actually has been done is to build up to a solution step by step, eliminating as many nodes as possible along the way. It is obvious that for the final step one can have the B boundary nodes of the system each connected by a primary element to one of the B' boundary nodes ($B' \leqslant B$) of a pseudo-element representing the entire remainder of the system, i.e., the entire system less the elements connecting points B with points B'.

It can be shown that this is so by considering the system as a whole and then reducing it, rather than by starting with a small subsystem and building up to the complete system. The first ganglion can then be chosen to be everything except those primary elements that have one end incident on a boundary node. If such a ganglion can not be

chosen, then there is more than one structurally independent system involved and they must be solved separately. This statement is not as trivial as it may seem to be at first reading. It is entirely possible that a piping network that would be considered as one system by reason of its application or fluid flow could actually be several independent structural systems. This could be caused by the pipe being rigidly anchored at one or more points thus effectively isolating the pipe on one side of the anchor from the effects of any change on the other side.

Although many of the boundary nodes of a piping system may be topologically identical (for example all nodes at which an element is rigidly connected to "ground") it is necessary to consider each as a separate entity because of the necessity of using different local coordinate systems and then different transformation matrices. Thus if two or more elements terminate on what is geometrically the same boundary node, they must be considered as terminating on topologically separate nodes. Therefore, there will be only one primary element incident on each boundary node. (See definition, page 7). Keeping this in mind it can be seen that the number of nodes connected to boundary nodes by primary elements must be less than, or at the most equal to B. Thus for the reduced system N is less than, or at most equal to 2B. Taking the limiting case, N = 2B, the node stiffness matrix K_N will be 12B x 12B and the frequency determinant will be (12B-P) x (12B-P) where P lies in the range $0 \leq P \leq 6B$. If all the boundary nodes represent complete constraints in all degrees of freedom P will be 6B, the other limit being no constraint on the boundaries and P equaling zero. Thus the order of the frequency determinant will be between 6B x 6B and 12B x 12B. The

restriction of having only one primary element incident on a boundary node would not be necessary for the solution of a mathematically similar problem where physical location and metric geometry are of no importance, for example, an electrical network.

3.3 Reduction of Parallel Elements

To provide a tool for future computational work, and to demonstrate the use of topological matrices it will be shown that parallel elements, i.e., primary elements connecting the same pair of primary nodes, can be reduced to an equivalent element. The stiffness matrix of this equivalent element will be the sum of the stiffness matrices of the various component primary elements. This is not a demonstration of the reduction method of section 3.2 as there are no nodes eliminated. The reduction of two parallel elements into one equivalent element is shown here. The logical extension is then made that this equivalent element can be taken parallel with another primary element and these may be reduced to a second equivalent element. As many parallel elements as required can be handled in this manner.

Take the case of two parallel elements, Fig. 3.3.1.

Fig. 3.3.1

1 and 2 are nodes, and a,b,c, and d are element ends.

$$Z_a = U_{ba}\ Z_b \qquad \text{from which} \quad F_{a,b} = K_{ab}\ D_{a,b}$$

$$Z_c = U_{dc}\ Z_d \qquad \text{from which} \quad F_{c,d} = K_{cd}\ D_{c,d}$$

After transforming these to some global coordinate system the primitive stiffness matrix can be assembled, viz.:

$$F_E = K_p D_E$$

$$\begin{bmatrix} F_a \\ F_b \\ F_c \\ F_d \end{bmatrix} = \begin{bmatrix} K_{ab} & 0 \\ 0 & K_{cd} \end{bmatrix} \begin{bmatrix} D_a \\ D_b \\ D_c \\ D_d \end{bmatrix} \qquad 3.3.1$$

Generating the topological matrices

$$A^T = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} I & 0 & I & 0 \\ 0 & I & 0 & I \end{bmatrix} \end{array}$$

$$A = \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \\ 0 & I \end{bmatrix}$$

and since $K_N = A^T K_p A$

$$K_N = \begin{bmatrix} I & 0 & I & 0 \\ 0 & I & 0 & I \end{bmatrix} \begin{bmatrix} K_{ab11} & K_{ab12} & 0 & 0 \\ K_{ab21} & K_{ab22} & 0 & 0 \\ 0 & 0 & K_{cd11} & K_{cd12} \\ 0 & 0 & K_{cd21} & K_{cd22} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \\ 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} I & 0 & I & 0 \\ 0 & I & 0 & I \end{bmatrix} \begin{bmatrix} K_{ab11} & K_{ab12} \\ K_{ab21} & K_{ab22} \\ K_{cd11} & K_{cd12} \\ K_{cd21} & K_{cd22} \end{bmatrix}$$

$$= \begin{bmatrix} K_{ab11} + K_{cd11} & K_{ab12} + K_{cd12} \\ K_{ab21} + K_{cd21} & K_{ab22} + K_{cd22} \end{bmatrix}$$

$$= K_{ab} + K_{cd} \qquad 3.3.2$$

Since $F_N = K_N D_N$

it has been shown that

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} K_{ab} + K_{cd} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \qquad 3.3.3$$

Thus, the equivalent stiffness matrix for two parallel elements is the sum of the two individual stiffness matrices. From the above we can see that the stiffness matrix for n parallel elements would be the sum of the n individual stiffness matrices.

This intuitively attractive result could probably have been shown without resorting to the topological matrices. The preceding derivation, however, puts into a definite mathematical form what would have to be understood in another derivation. That is, the sum of the forces on the node equals the sum of the forces at the element ends and the displacements of all element ends at any node are identically equal.

An equivalent element with its own stiffness matrix has now been generated, and may be used as one of the building blocks of the system. However, in a manner equivalent to reversing what we did to get a stiffness matrix from a transfer matrix in section 2.1 we can get, if desired, a transfer matrix for this equivalent element, in whichever local coordinate system is convenient. If the system is so configured that after the reduction of the parallel elements the equivalent element is then one

of two or more elements of a primary path, the transfer matrix may then be used in the generation of a primary element including the equivalent element.

CHAPTER IV

COMPUTER PROGRAM PLANNING

4.1 Discussion

It should be obvious by now that the solution of any but the simplest problems by these methods will require the use of a high speed digital computer, and that the use of intermediate tapes or auxiliary memory will most likely be required.

The primary object of all the calculating is to find some or all of the real values of frequency which will make the frequency determinant go to zero. The corresponding "mode shapes" can easily be computed by a repeated pass using the correct frequency. Since the computer can only calculate with numbers, it will not be possible to set up an equation for the determinant and solve it exactly. It will not even be possible to set up an equation and try to solve it by substituting very many values of frequency. Rather, all calculations must be done numerically using assumed values of frequency. The frequencies which make the frequency determinant equal to zero must be found by an iterative process. This means that virtually all the calculations must be redone for each assumed frequency.

The writing of a computer program to do the work involved is not a simple matter. The ideal program should be written in a widely used engineering language (such as FORTRAN) and arranged to allow data to be presented in a most expeditious manner. A good example of this type of program is "STRESS" by Fenves et. al. [4] [5]. "STRESS" solves static force-displacement problems by use of a stiffness matrix method.

No attempt has been made to write a program for this thesis. However, the outline of such a program is shown in Fig. 4.1.1. A method for systematically approaching this problem is given in section 4.2.

The reader interested in actually programming this type of problem is advised of the existance of a FORTRAN program written by Fink [6] in 1964. Program VIPIPE is severely restricted in the size and topological configuration of the systems it can handle. It was written to determine in-plane and out-of-plane vibrations of a two dimensional piping system and does not make use of topological matrices. It would be a good starting point for the generation of a more sophisticated program and many of its subroutines, in particular those for calculating transfer matrices, could probably be put to good use.

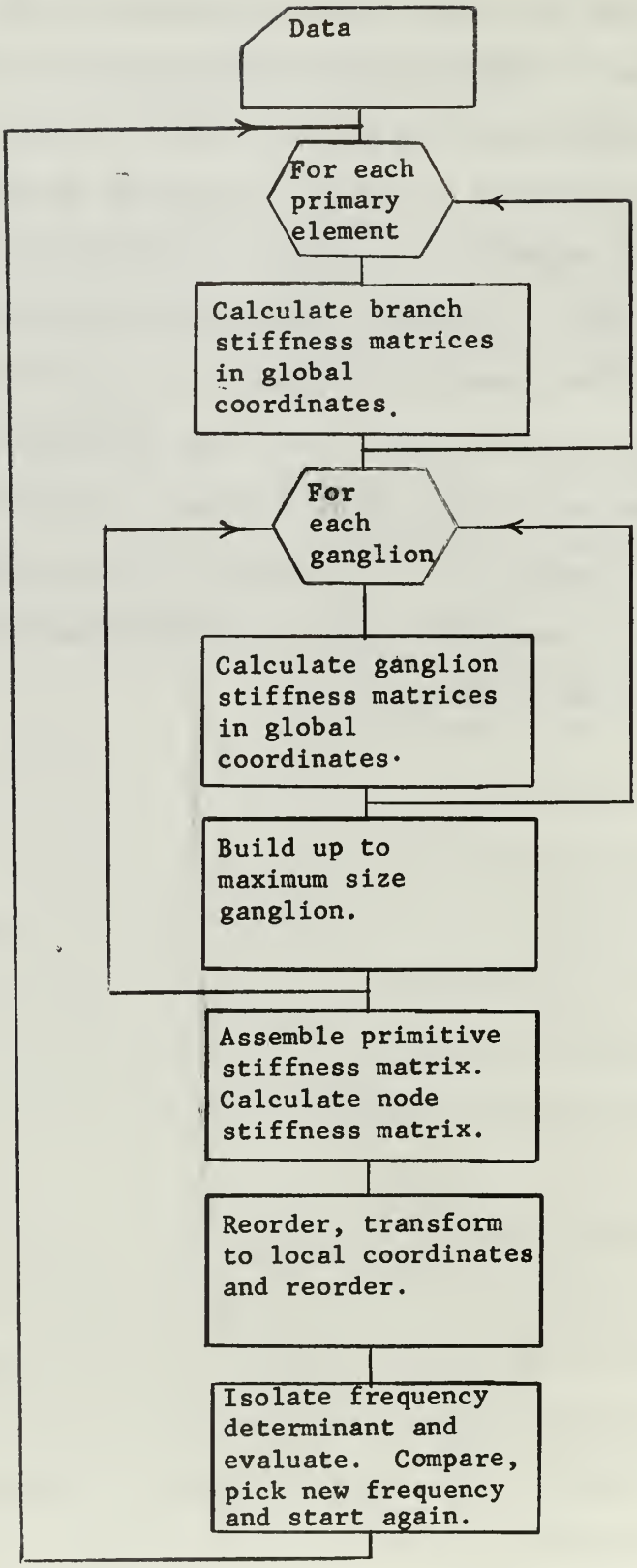

Data
For each primary element
Calculate branch stiffness matrices in global coordinates.
For each ganglion
Calculate ganglion stiffness matrices in global coordinates.
Build up to maximum size ganglion.
Assemble primitive stiffness matrix. Calculate node stiffness matrix.
Reorder, transform to local coordinates and reorder.
Isolate frequency determinant and evaluate. Compare, pick new frequency and start again.

Figure 4.1.1

4.2 Expansion of Computational Concepts

The purpose of this section is to develop to a greater extent the approach to a computer solution for a general problem of this category. The basic flow diagram for a computer program shown in Fig. 4.1.1 will be used as a rough guide.

The following assumptions are made:

a. A computer memory of about 30K available cells is to be utilized.

b. Intermediate tapes rather than drums or discs will be used. This assumption puts some constraints on the intermediate storage of information as binary tapes cannot be randomly accessed whereas drums or discs can.

c. Most of the calculating will be done with double precision arithmetic.

d. The system to be solved may be of any size.

The following systematic approach is recommended.

1.) Display the system topologically showing only the boundary and primary nodes and the primary elements connecting them. Number the nodes, using the lowest numbers for the boundary nodes, and number the primary elements.

2.) Determine the way in which the system will be divided into ganglia and subsequently recombined. A procedure for making cuts for an "optimum" reduction is beyond the scope of this work, and indeed perhaps the idea of an optimal reduction is frivolous. However, certain operations should be readily apparent, and the first of these is the

combining of parallel elements, where present. From this point forward it is an empirical process. One must use his judgement in working toward replacing the entire system, except for the boundary nodes and the primary elements incident on the boundary nodes, with a single pseudo-element. For reasons that will be discussed later, it is best to set up two or more different sets of ganglia so that the problem may be solved more than once and the answers compared.

3.) Display the system so that the geometry will be readily available for computer programming. An isometric line drawing, (not necessarily to scale, but at least reasonably proportional to the actual layout) will probably be adequate.

4.) Analyze the boundary node constraints. It is necessary that each node be either unrestrained or rigidly restrained in each of its six degrees of freedom. If this is not the case then an imaginary element with characteristics equivalent to the desired boundary constraints must be added between the element end and the boundary node to satisfy the above condition.

5.) Analyze each other node. The assumption made in this method is that elements are rigidly connected together at the nodes. If this is not the case, an imaginary spring element of desired characteristics must be included wherever required.*

6.) Establish an origin of coordinates and an orthogonal set of global coordinates.

---

*This is related to the difficult problem of releases. See reference [7].

7.) Determine and tabulate for each element the following data:

a. Type of element

1. straight pipe
2. arc of circle pipe
3. "imaginary" element
4. non-pipe (hanger)

b. weight per unit length of pipe

c. weight per unit length of fluid and insulation

d. pipe diameter

e. wall thickness

f. E (modulus of elasticity)

g. G (modulus of elasticity in shear)

h. coordinates of ends and of center of curvature (if an arc)

i. radius of curvature

8.) Read in the data for each element; make whatever calculations are necessary to convert the raw data into an efficient form for future calculating; and store all this information on tape. All this data is independent of frequency, and must be used in each iteration to calculate transfer matrices. It is thus vital that the information be stored in the order that it is to be used so that the tape may be used in the most expeditious manner. Let us assume that we have a set of data cards carrying all the information for all the elements of each primary element. We also have a larger set composed of all these sets, so that the data is on cards for all elements of the system. Now go back to 2) and

pick the first ganglion to be reduced. Decide on the order in which the primitive stiffness matrix is to be assembled. (This will also be a good time to determine the way in which the node stiffness matrix is to be ordered and to generate the topological matrices for this ganglion, though these items will not be needed for a while.) Assemble the data cards for each primary element in order from one end to the other. Remember that the coordinates of the first end of the first element will determine the local coordinate system in which that particular primary element will be calculated. For each primary element set there should be an initial card indicating the numbering of elements in the primary element. Assemble all the primary element card sets in the order in which the primitive stiffness matrix is to be assembled.

9.) Choose the next reduction to be made. If it is independent of the primary elements already assembled for reading, proceed as above. If it contains the ganglion (or several ganglia) assembled for reading, proceed as above only for the primary elements not already used. If it is composed entirely of elements whose cards are already assembled, no action is required at this time. (It would still be good at this time to set up the order for the node stiffness matrix and generate the topological matrices for this ganglion).

10.) When the ordering is complete the element data is ready to be read in. A flag indicating that no more element data is to be read should be added. The number of elements in each primary element are stored in an array to facilitate retrieval of information from the tape. The decision on exactly how to handle data, particularly in reference to the amount

of data to be read in and transferred to tape at one time must be left to the programmer. It is faster to accumulate data in core and transfer it all to tape with one WRITE TAPE statement than to go to tape with each individual set as it is calculated. It should be noted that much economy in data card preparation can be obtained by eliminating the necessity for duplicating information that is identical for many elements, i.e., Young's modulus, diameter, etc.

11.) Now starting with the first ganglion to be reduced, read in the appropriate topological matrix (A or A^T) and store it. It will not be necessary to have both A and A^T in memory since by programming we can utilize the stored matrix either as itself or as its transpose. Since all elements of the topological matrices are either $\delta_{6}I$ or $\delta_{6}0$ great storage economy can be obtained by representing these 6x6 elements by a Boolean 1 or 0 respectively and storing thirty-two of these per core memory word.* It will probably not be necessary to go out on tape with these matrices.

At this point all the data that will be constant from iteration to iteration should be either on tape or in core.

12.) With the assumption of an initial frequency the first iteration may now be started. From this point on, all calculations will be done with double precision arithmetic. Start by taking the first ganglion as previously ordered, and read back into the core all the data relevant to the elements of this ganglion. By the use of sub-routines the stiffness matrix for each primary element of the ganglion is calculated. It will

*The number of logical elements that may be stored in one word varies from computer to computer. The number thirty-two used here applies to the CDC 1604.

not be necessary to store the numerous transfer, rotation and transformation matrices that are generated for if they are generated in the correct order and multiplied together then all that need be stored is the product. The stiffness matrix for each primary element will occupy 2 x 12 x 12 = 288 core storage locations where the 2 results from double precision arithmetic. Now although the primitive stiffness matrix, K_p, is shown as a large square matrix, it will be assembled in the computer as a three dimensional array. The reason for this is that K_p consists only of diagonal elements, each of which is of order 12 x 12, and all the other elements are zero. By using a three dimensional array we can store only the diagonal elements and not waste space storing zeros. The storage required for K_p will be 288 x M'/2, and if the stiffness matrices have been cleverly stored when they were generated K_p already exists. The node stiffness matrix is now generated from $K_N = A^T K_p A$. Note that the column vectors of node forces and displacements are never actually used in the solution. From K_N we generate $K_{N'}$ as shown in equations 3.1.1 and 3.1.2. The order of this matrix will be 6B' x 6B'; and the number of memory words necessary to hold it will be 2 x 6B' x 6B'. As before, the 2 represents the requirement of two words per item in double precision arithmetic.

Assuming that this reduced ganglion will be used as part of the next ganglion to be reduced, we will plan to leave $K_{N'}$ in core memory. If it is not to be used for a while, and if space is at a premium for other calculations, it may be better to store $K_{N'}$ on tape.

13.) By continuing in a similar manner eventually we will arrive at the largest ganglion, namely, that one including everything except

the primary elements incident upon the boundary nodes. The order of the resultant stiffness matrix for this ganglion will be 6B x 6B. Using this stiffness matrix, and the stiffness matrices of the remaining primary elements we now set up the primitive stiffness matrix for the entire system and by pre- and post-multiplying it by the topological matrices obtain the node stiffness matrix for the entire system. Since we have assembled these matrices carefully, the reordering described in section 2.h will not be necessary. The transformation back to local coordinates is made for the boundary nodes. With the boundary nodes represented in local coordinates it is now possible to make the second reordering as described in section 2.i. The frequency determinant is now isolated and its value calculated. This value and the corresponding value of frequency at which the calculations have been made are saved.

14.) A new value of frequency is now assumed and the above process repeated. Subsequent values of the determinant may be used in conjunction with the values found previously to predict the next frequency to be tried in order that the calculations may converge on the desired natural frequency. Alternatively, calculations may be made at constant intervals and the results plotted.

4.3 Computational Difficulties

There is one predominant problem that keeps appearing, if only implicitly, in work with matrices. Any time matrix equations are manipulated in such a way that the inverse of some matrix is required, the assumption must be made that the matrix to be inverted is non-singular. Matrices are inverted many times in the course of a solution by the methods discussed herein, and the possibilities of having singular matrices occur

occasionally are significant and somewhat disturbing.

In the determination of the stiffness matrix of an element from its transfer matrix it is necessary to invert a 6 x 6 matrix as described in section 2.b.

$$\begin{bmatrix} D_m \\ F_m \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} D_k \\ f_k \end{bmatrix} \qquad \text{2.2. (repeated)}$$

from which

$$K = \begin{bmatrix} U_{12}^{-1}U_{11} & -U_{12}^{-1} \\ U_{21} - U_{22}U_{12}^{-1}U_{11} & U_{22}U_{12}^{-1} \end{bmatrix}$$

Note that U_{12} is the 6 x 6 matrix to be inverted. As some or all of the elements of U_{12} are frequency dependent (all other parameters are dependent upon physical constants and are thus fixed) it can be seen that there will be some values of frequency for which the determinant of U_{12} will vanish. These frequencies correspond to the natural frequencies of the element with both ends "built-in" or cantilevered. In matrix form, from 2.2.

$$D_m = U_{11}D_k + U_{12}f_k \qquad D_m = D_k = 0$$

$$0 = 0 + U_{12}f_k$$

and therefore $\left|U_{12}\right| = 0$. Only the real values of frequency that satisfy this relationship are of interest.

What this means in terms of programming is that whenever the assumed value of frequency at which a series of calculations is being made is the same as (or close to) one of the natural frequencies of one of the primary elements it will not be possible to generate the stiffness matrix for that element. It will be necessary to assume a new frequency near the

unacceptable one and to try to calculate to a conclusion again.

A similar problem may arise in the calculation of the equivalent stiffness matrix for a ganglion. Assuming that stiffness matrices have been satisfactory calculated for each primary element of the ganglion as shown in the arithmetical manipulations of equations 3.1.1, it is still necessary to invert a matrix and this matrix, also being made up of elements dependent upon the frequency, may become singular. The frequencies at which this matrix becomes singular correspond to the natural frequencies of a system equivalent to the ganglion but with the boundary nodes rigidly fixed.

The inability to calculate to a conclusion due to an inability to invert part of the node stiffness matrix of a ganglion can be avoided, in most cases, by breaking the system up differently and thus using different ganglia. It is for this reason that in section 4.2 it is recommended that any problem of this type be solved using several different combinations of ganglia. Any reasonably sophisticated computer program will, of course, provide some sort of diagnostic message and proceed to calculate at a different value of frequency whenever some matrix cannot be inverted.

The problem of a singular matrix affecting the calculations should not be too serious at the low frequency end of the vibrational spectrum. It can reasonably be expected that the natural frequencies of a ganglion will be considerably greater than the lowest natural frequency of the overall system. It is not possible to forecast at what frequency problems will actually begin to occur. What is to be done is just to proceed with the calculations, avoiding those frequencies at which calculations cannot be made, but calculating as closely as possible on both

sides of such frequencies.

There are other problems involved in the creation of a computer program capable of handling large linear systems by matrix methods. There is, for example, the possibility that even after all reductions have been made the frequency determinant will still be too large for core memory. There will also be the need for making a number of tests during the course of a program run, both for debugging purposes and to provide a hedge against gross errors. (An example of this would be the testing of the value of mass of an element to insure that it is positive). It is recommended therefore that the "team" for developing this program consist of at least a mechanical engineer well versed in vibrations and a computer programmer with experience in handling large systems and matrices.

BIBLIOGRAPHY

1. BAIRD, W. S. Jr., Vibration Analysis of Three Dimensional Piping Systems with General Topology. Master's Degree Thesis, U. S. Naval Postgraduate School, 1965.

2. PESTEL, E. C. and LECKIE, F. A., Matrix Methods in Elastomechanics. McGraw-Hill Book Company, Inc., 1963.

3. FENVES, S. J. and BRANIN, F. H. Jr., A Network-Topological Formulation of Structural Analysis. Technical Report #00.979, Feb. 25, 1963. IBM Data Systems Division Development Laboratory, Poughkeepsie, N. Y.

4. FENVES, S. J., et al. Stress: A Reference Manual M.I.T. Press, 1965.

5. FENVES, S. J. et al. Stress: A User's Manual M.I.T. Press, 1964.

6. FINK, G. E. Vibration Analysis of Piping Systems. Master's Degree Thesis, U. S. Naval Postgraduate School, 1964.

7. FENVES, S. J., MAUCH, S. P., and KINRA, R. K., Treatment of Releases and Constraints in the Network Formulation of Structural Analysis. Civil Engineering Studies; Structural Research Series No. 299. University of Illinois, Oct. 1965.

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A theoretical method for representing the physical characteristics of a portion of a mechanical piping system in terms of a stiffness matrix involving only those joints at which the sub-system is connected into the overall system is presented herein. The use of this method in reducing the size of the matrices that are needed for finding the normal vibration frequencies of a piping system is discussed, and an outline for a digital computer program to use this method is presented.

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